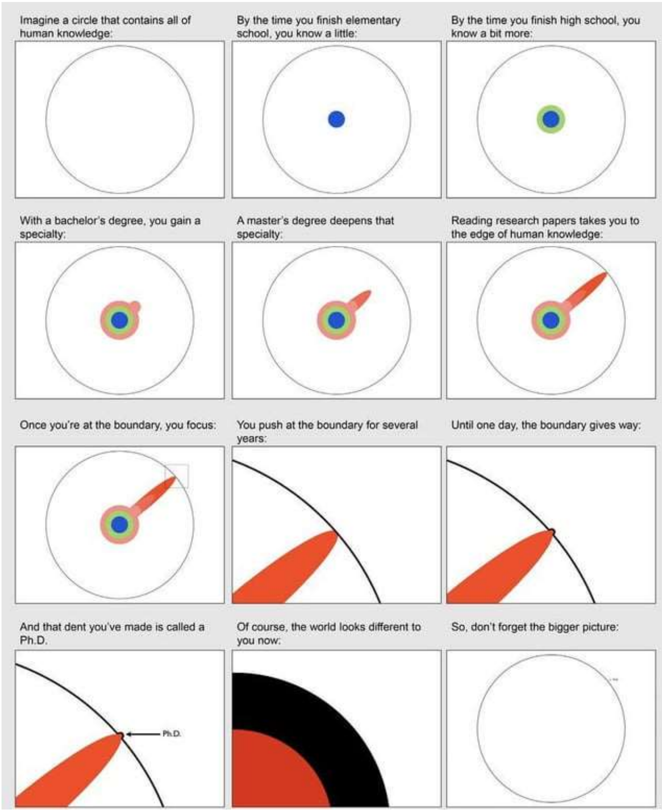
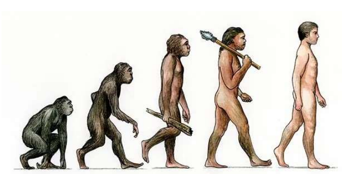
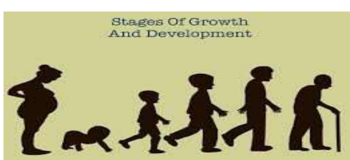
Real Analysis in Stat 6410 (Iowa State)

Wednesday, June 26, 2024 3:06 PM







More than just a catchy phrase, "ontogeny recapitulates phylogeny" is the foundation of recapitulation theory. Recapitulation theory posits that the development of individual organisms (ontogeny) follows (recapitulates) the same phases of the evolution of larger ancestral groups of related organisms (phylogeny). Feb 10, 2017

NYBG New York Botanical Garden
https://www.nybg.org › blogs › science-talk › 2017/02

A Catchy Phrase, But is It True? - New York Botanical Garden

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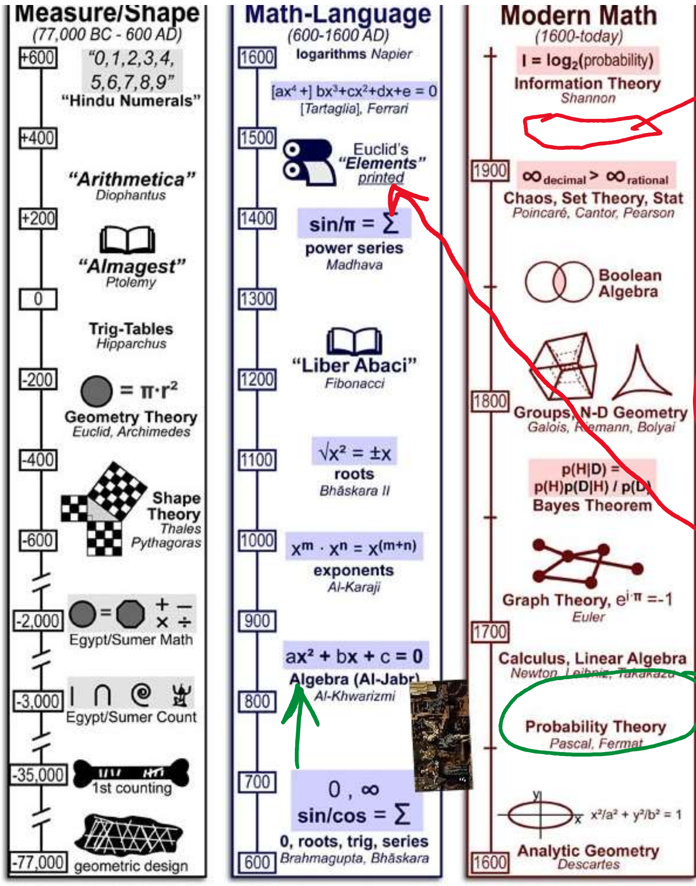
Measure theory



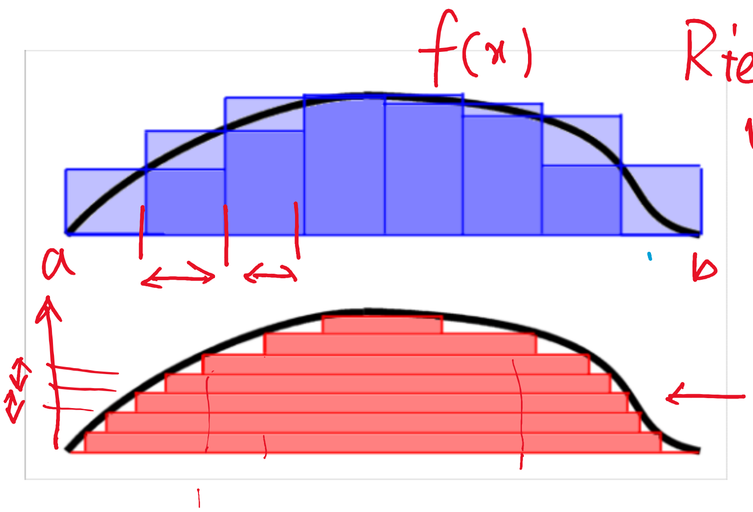
measure theory

printing press invented.

Stat 6410 : measure theory

- need basics of analysis / calculus.

- extends those ideas to stable math foundation for prob. theory.



Riemann integral (usual)

$$\int_a^b f(x)\,dx$$

Lebesgue integral $\int_{[a,b]} f\,d\lambda$

λ : lebesgue meas.

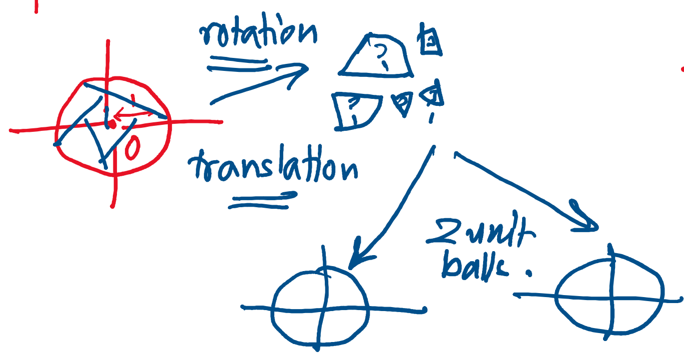
Intro to measure theory

(everything here will be discussed in detail later)

* Shortcomings of Riemann integration / Calculus.
* Connecting loose ends of prob. theory !
* Math paradoxes that lead to unreasonable results

* Math paradoxes that lead to unreasonable results

eg:



Banach-Tarski paradox

(Reasons for Measure Theory)

$(\Omega, \mathcal{F}, P) \xrightarrow{X_i} (\mathbb{R}, \mathcal{B}(\mathbb{R}))$

Theorem 5.5.9 (Strong Law of Large Numbers) *Let $X_1, X_2, \ldots$ be iid random variables with* $\mathrm{E}X_i = \mu$ *and* $\mathrm{Var}\, X_i = \sigma^2 < \infty$, *and define* $\bar{X}_n = (1/n)\sum_{i=1}^n X_i$. *Then, for every* $\epsilon > 0$,

$$P\left(\lim_{n\to\infty} |\bar{X}_n - \mu| < \epsilon\right) = 1; \quad \longleftrightarrow \quad P\left(\lim_{n\to\infty} |\bar{X}_n - \mu| = 0\right) = 1$$

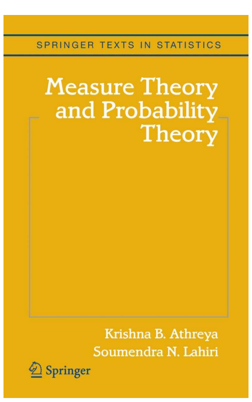
$P(\{\omega : \lim_{n\to\infty} |\bar{X}_n(\omega) - \mu| = 0\}) = 1$

or $\bar{X}_n - \mu \to 0$ a.e (P)

that is, $\bar{X}_n$ converges almost surely to μ.

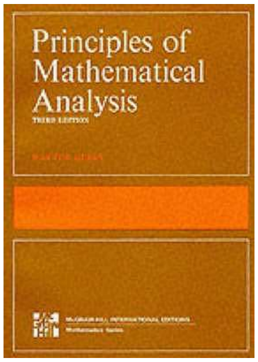
Usual text for 641

https://www.amazon.com/Measure-Theory-Probability-Springer-Statistics/dp/038732903X



Classic reference for analysis (advanced calculus):

https://www.google.com/search?client=opera&q=analysis+rudin&sourceid=opera&ie=UTF-8&oe=UTF-8



Rudin's textbook for real analysis.